Consider the region defined by $y \le x+1$, $y \ge 2x-3$ and $x \ge 1$.

$x+1 = 2x-3$

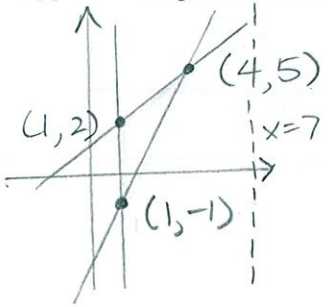
$4 = x \rightarrow y = 5$

$x = y-1$, $x = \frac{1}{2}(y+3)$

SCORE: ____ / 12 PTS

[a] If the region is revolved around the line $x = 7$, write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

[i] using the disk or washer method **(NOTE: You do NOT need to simplify your integrand.)**



$$\left[\int_2^5 \pi \left[(7-(y-1))^2 - (7-\tfrac{1}{2}(y+3))^2\right] dy\right] + \left[\int_{-1}^2 \pi \left[(7-1)^2 - (7-\tfrac{1}{2}(y+3))^2\right] dy\right]$$

(each π: $\frac{1}{2}$; each $(7-1)^2$: $\frac{1}{2}$; each $(7-\frac{1}{2}(y+3))^2$: $\frac{1}{2}$)

[ii] using the shell method **(NOTE: You do NOT need to simplify your integrand.)**

$$2\pi \int_1^4 (7-x)(x+1-(2x-3))\, dx$$

(1) POINT EACH EXCEPT AS NOTED

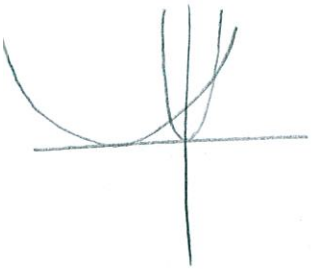
[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $x-$ axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{\pi}{8} \int_1^4 (x+1-(2x-3))^2\, dx$$

($\frac{1}{2}$)

Find the area between the curves $y = 4x^2$ and $y = (x+3)^2$ over the interval $-2 \le x \le 2$. **SCORE: ____ / 6 PTS**

NOTE: The answer is NOT 20 NOR 4.



$$4x^2 = (x+3)^2$$
$$4x^2 = x^2 + 6x + 9$$
$$3x^2 - 6x - 9 = 0$$
$$3(x-3)(x+1) = 0$$
$$x = 3, -1$$

(1) POINT EACH

$$\int_{-2}^{-1} (4x^2 - (x+3)^2)\,dx + \int_{-1}^{2} ((x+3)^2 - 4x^2)\,dx$$

$$= \int_{-2}^{-1} (3x^2 - 6x - 9)\,dx + \int_{-1}^{2} (-3x^2 + 6x + 9)\,dx$$

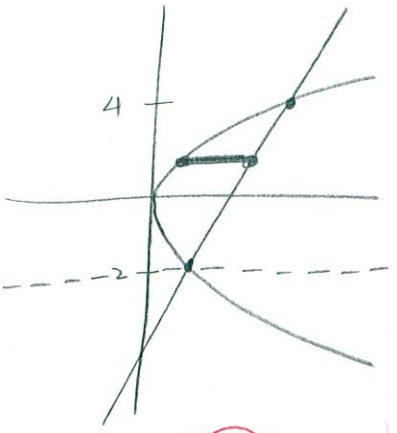
$$= (x^3 - 3x^2 - 9x)\Big|_{-2}^{-1} + (-x^3 + 3x^2 + 9x)\Big|_{-1}^{2}$$

$$= ((-1 - 3 + 9) - (-8 - 12 + 18)) + ((-8 + 12 + 18) - (1 + 3 - 9)) = 5 + 2 + 22 + 5$$

$$= 34$$

The region bounded by $y^2 = 4x$ and $y = 2x - 4$ is revolved around the line $y = -2$. **SCORE: ____ / 12 PTS**

Find the volume of the resulting solid. **(NOTE: Your final answer must be a number, not an integral (or sum of integrals).)**



$x = \frac{1}{4}y^2 \quad x = \frac{1}{2}y + 2$

$\frac{1}{4}y^2 = \frac{1}{2}y + 2$

$y^2 = 2y + 8$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0 \rightarrow y = 4, -2$

(2) $\int_{-2}^{4} 2\pi (y - -2)(\frac{1}{2}y + 2 - \frac{1}{4}y^2)\,dy$ (1/2)

$= 2\pi \int_{-2}^{4} (2+y)(2 + \frac{1}{2}y - \frac{1}{4}y^2)\,dy$ (1/2) (2)

$= 2\pi \int_{-2}^{4} (4 + 3y - \frac{1}{4}y^3)\,dy$ (2)

$= 2\pi \left(4y + \frac{3}{2}y^2 - \frac{1}{16}y^4\right)\Big|_{-2}^{4}$ (1/2)

$= 2\pi (16 + 24 - 16 - (-8 + 6 - 1))$

$= 2\pi (24 - -3)$

$= 54\pi$ (1/2)

TALK TO ME IF YOU USED WASHER METHOD (dx)